NAME ___________________________________ COMMON CORE GEOMETRY

Module 3

Extending to Thee Dimensions

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2/13	10-12	Quiz Demonstration of 3-D terminology	No homework
2/23	10-12	Lesson 5: Three Dimensional Space	Worksheet Lesson 5
2/24	12-13	Lesson 6: General Prism and Cylinders and Their Cross-Sections	No homework
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2/26	14-16	Lesson 7: General Pyramids and Cones and Their Cross-Sections	No homework
2/27	17-18	Lesson 8: Definition and Properties of Volume	Worksheet Lesson 8
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3/3	19-21	Lesson 9: Scaling Principle for Volumes	Worksheet Lesson 9
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3/5	24-25	Lesson 11: The Volume Formula of a Pyramid and Cone	Worksheet Lesson 11
3/6	24-25	Lesson 11 Cont. Quiz	No homework
3/9		Review	Review sheet & Study
3/10		Test	

Common Core High School Math Reference Sheet
(Algebra I, Geometry, Algebra II)

CONVERSIONS

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilograms
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

FORMULAS

Triangle	$A = \frac{1}{2}bh$	Pythagorean Theorem	$a^2 + b^2 = c^2$
Parallelogram	$A = bh$	Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Circle	$A = \pi r^2$	Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Circle	$C = \pi d$ or $C = 2\pi r$	Geometric Sequence	$a_n = a_1 r^{n-1}$
General Prisms	$V = Bh$	Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Cylinder	$V = \pi r^2 h$	Radians	$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$
Sphere	$V = \frac{4}{3}\pi r^3$	Degrees	$1 \text{ degree} = \frac{\pi}{180} \text{ radians}$
Cone	$V = \frac{1}{3}\pi r^2 h$	Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$
Pyramid	$V = \frac{1}{3}Bh$		

Lesson 1: What Is Area?

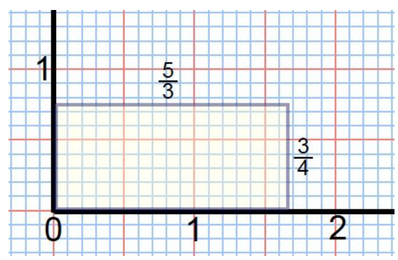
Exploratory Challenge 1

a. What is area? Space that is ___________ a closed figure.

b. When would you need to find the area of a figure? ____

c. What is the area of the rectangle below whose side lengths measure $3\ units$ by $5\ units$?

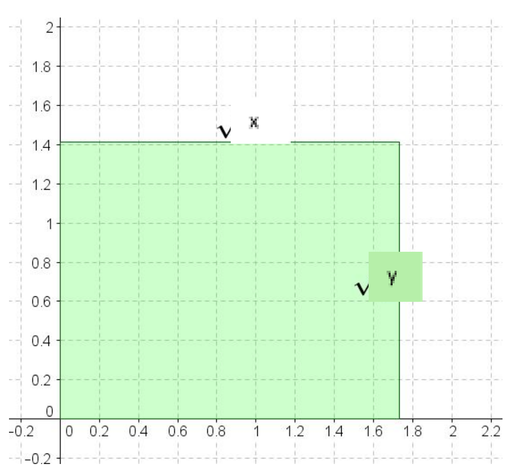
d. What is the area of the $\frac{3}{4} \times \frac{5}{3}$ rectangle below?



Exploratory Challenge 2

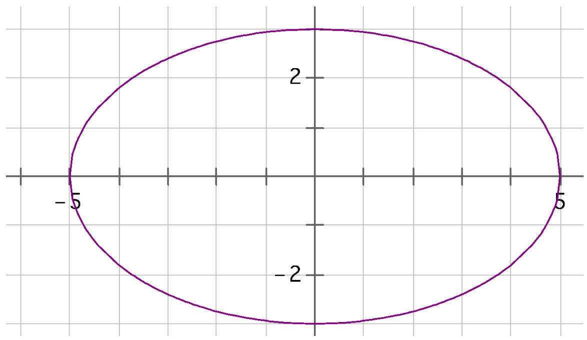
a.) What is the area of the rectangle below whose side lengths are unknown? Use the unit squares on the graph to guide your approximation. Explain how you determined your answer

b.) Is your answer precise?



Discussion:

1. If it takes one can of paint to cover a unit square in the coordinate plane, how many cans of paint are needed to paint the region within the curved figure?



a.) Color in the unit squares that we have

b.) How many are there?

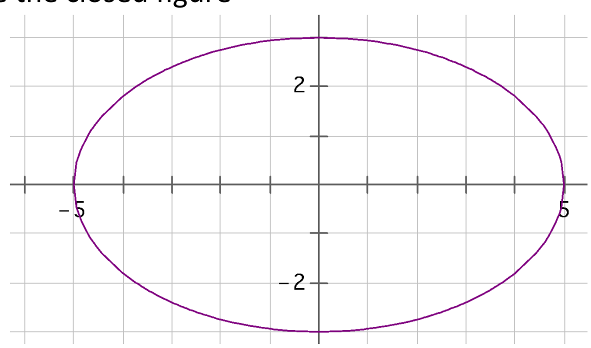
c.) Color in the half squares

d.) How many full squares (2 halves) do we have?

e.) What is the minimum number of squares inside the shape?

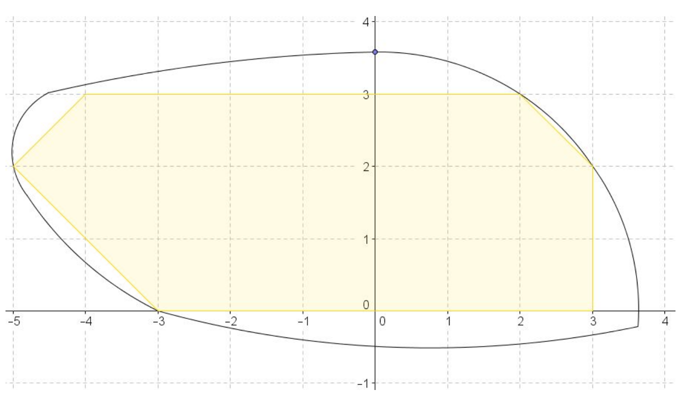
a.) Color full OR half squares that lie partly OR fully inside the closed figure

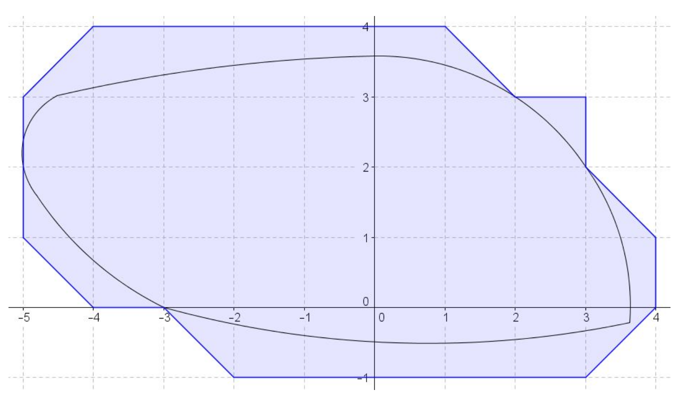
b.) How many are there in total?



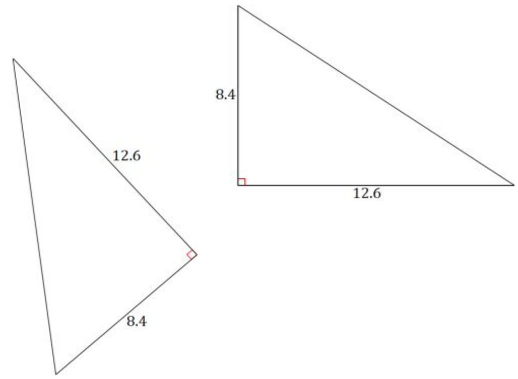
c.)Since both answers are either too much and too little what strategies could we use?

2.) Approximate the area of the figure shown below, the diagram on the left shows an underestimate of the area and the figure on the right shows an overestimate of the area of the figure.





Lesson 2: Properties of Area



Exploratory Challenge/Exercises 1–4

1.)Two triangles are shown below.

a. What do you know about the two triangles? Justify your answer

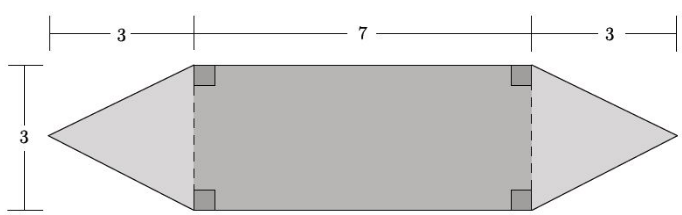
b. Which rigid motion(s) occurred?

c. Calculate the area of each triangle.

REVIEW SOME AREA FORMULAS:

Area of a Square	Area of a Triangle
Area of a Circle	Area of a Rectangle

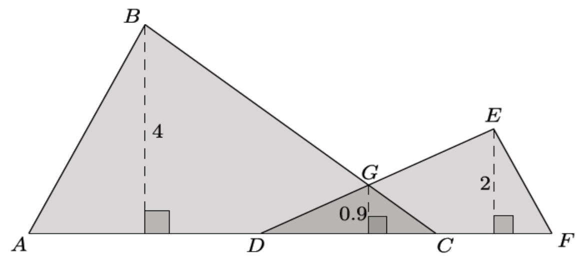
2.) Calculate the area of the shaded figure below.



Plan:

3.) Two triangles $\triangle ABC$ and $\triangle DEF$ are shown below. The two triangles overlap forming $\triangle DGC$. The base of figure $ABGEF$ is comprised of segments of the lengths lengths : $AD = 4$, $DC = 3$, and $CF = 2$. Calculate the area of the figure $ABGEF$.

Plan:

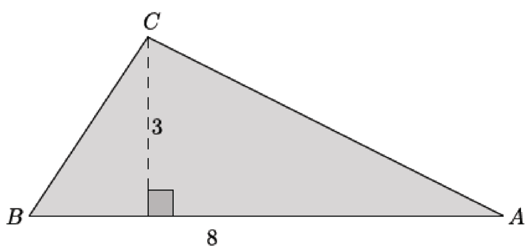


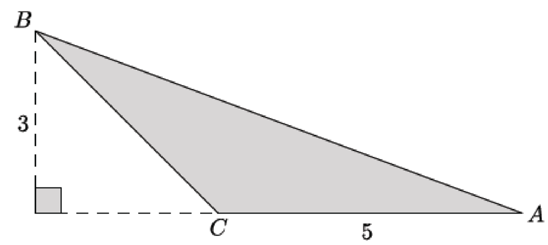
Lesson 3: The Scaling Principle for Area

Use the shapes below to fill in the chart:

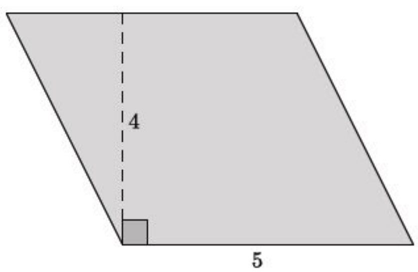
a.)

b.)

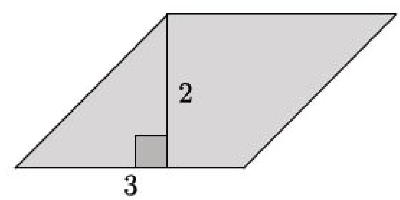




c.)



d.)



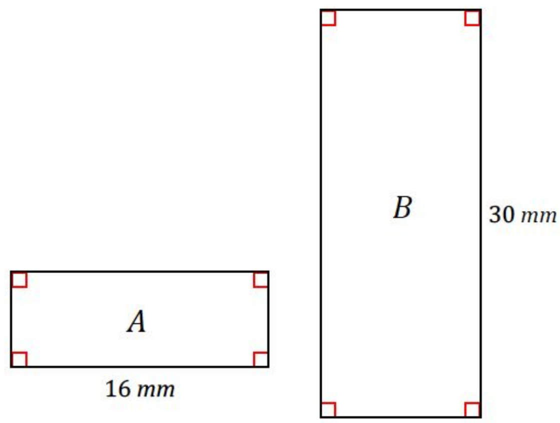
(i) Area of Original Figure	Scale Factor	(ii) Dimensions of Similar Figure	(iii) Area of Similar Figure	Ratio of Areas $Area_{similar} : Area_{original}$
	3			
	2			
	$\frac{1}{2}$			
	$\frac{3}{2}$			

d.) Make a conjecture about the relationship between the areas of the original figure and the similar figure with respect to the scale factor between the figures.

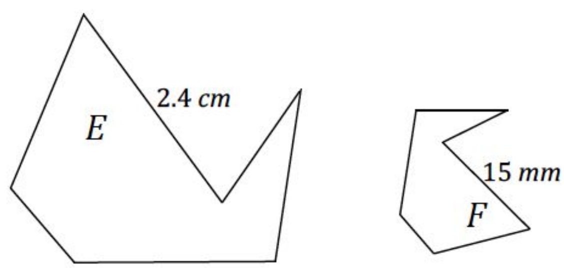
THE SCALING PRINCIPLE FOR AREA: IF ______________ FIGURES ARE RELATED BY A __________________ OF ______________, THEN THEIR RESPECTIVE ______________________ ARE RELATED BY A ____________________ OF __________________.

Exercises 1–2

Rectangles A and B are similar and are drawn to scale. If the area of rectangle A is $88\ mm^2$, what is the area of rectangle B?



2. Figures E and F are similar and are drawn to scale. If the area of figure E is $120\ mm^2$, what is the area of figure F?



Lesson 4: Proving the Area of a Disk

Opening Exercise

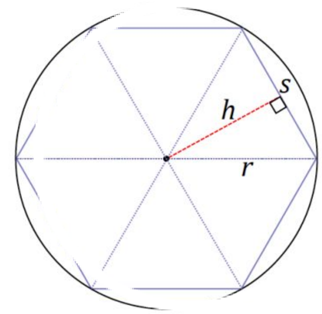
The following image is of a regular hexagon inscribed in circle C with radius r.

a.) Find a formula for the area of the hexagon in terms of the length of a side, s, and the distance from the center to a side.

Area of each triangle=

# of triangles (Area of each)=

If we rearrange the formula using commutative property we get:

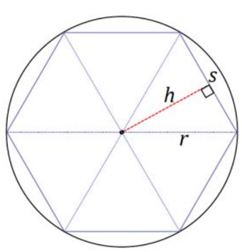


b.) Find the area of the hexagon if s=4 (What type of triangles are these?)

c.) By circumscribing a circle about the hexagon we can find an approximate area of the circle. Is this the best approximation? What could we do?

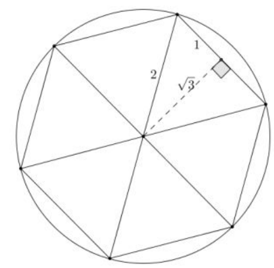
What is the formula for the Circumference (Perimeter) of a circle:___________

Rearrange the formula for finding the area:

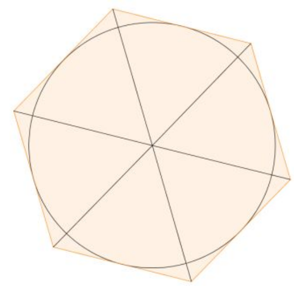


1.) You can find the approximate area of a **disk (A _________ object)** by:

-Approximate the area of a disk of radius 2 using an inscribed regular hexagon.



- Approximate the area of a disk of radius 2 using a circumscribed regular hexagon



1.)Based on the areas of the inscribed and circumscribed hexagons, what is an approximate area of the given disk?

2.) What is the area of the disk by the area formula, and how does your approximation compare?

3.) What could we do to the polygon to get a more accurate answer?

http://www.mathopenref.com/polygonregular.html

Lesson 5: Three-Dimensional Space

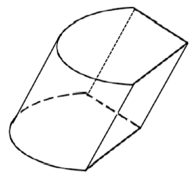
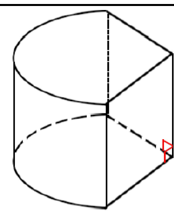
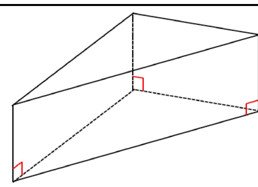
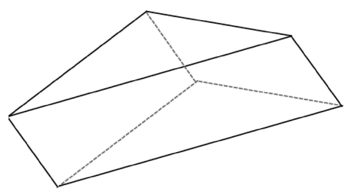
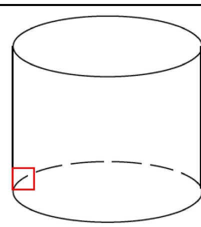
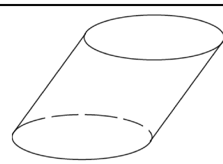
Opening Exercise

Sometimes working in 3-D can be hard to understand. Some student will be able to _____________ it in their heads, some are able to ___________ it on their paper and others will want to use_________________ to build it.

Words	Picture	Manipulative used
A point		
A plane		
A line *l* on a plane		
A line *k* that intersects a plane		

Exploratory Challenge- Work with partners.

Lesson 6: General Prisms and Cylinders and Their Cross-Sections

Figure and Description	Sketch of figure	Cross Section: A slice that is ________ to the ___________
General Cylinder		
Right General Cylinder		
Right Prism		
Oblique Prism		
Right Cylinder		
Oblique Cylinder		

Critical thinking:

How does a cylinder relate to a prism? Check it out!
http://www.mathopenref.com/cylinderprism.html

Fill in the blanks using the previous table and your knowledge of cylinders:

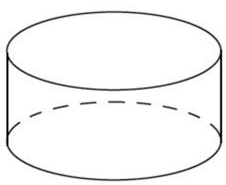
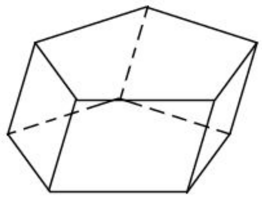
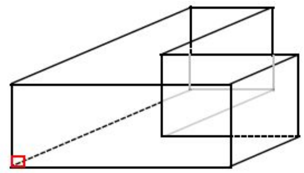
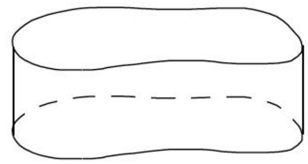
1.) A general cylinder with a polygon base is called ____.

2.) A general cylinder with a circular base is called a ____.

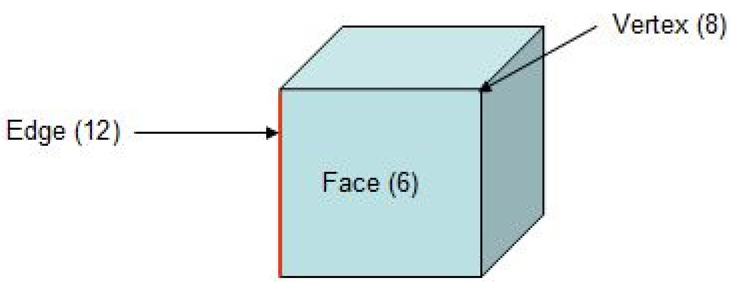
3.) A general cylinder with lateral edges perpendicular to the base is called a ___________________________.

4.) A general cylinder with lateral edges not perpendicular to the base is called a ________________________.

5.) Sketch the cross-section for the following figures:

a. 	b. 	c. 	d. 

6.) Consider the cube shown below, it shows the number of faces, verticies, and edges.



Find the number of Faces, verticies and edges for these shapes:

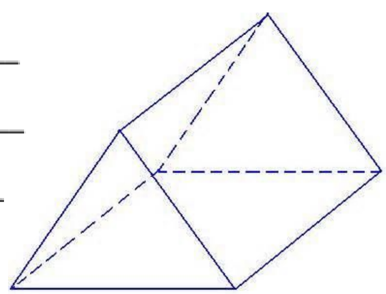
(Try this link! http://www.learner.org/interactives/geometry/3d_prisms.html)

TRIANGULAR PRISM

A.)Faces ___

Vertices____

Edges ____

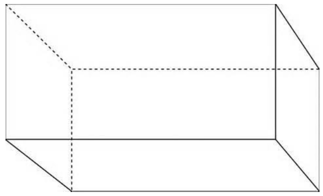


RECTANGULAR PRISM

B.)Faces ___

Vertices____

Edges ____

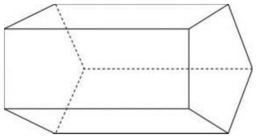


PENTAGONAL PRISM

C.)Faces ___

Vertices____

Edges ____

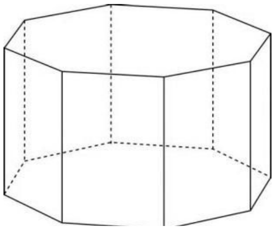


OCTOGONAL PRISM

D.)Faces ___

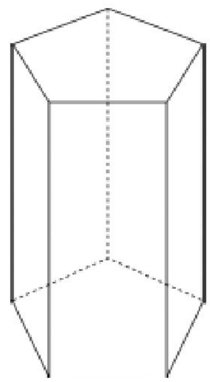
Vertices____

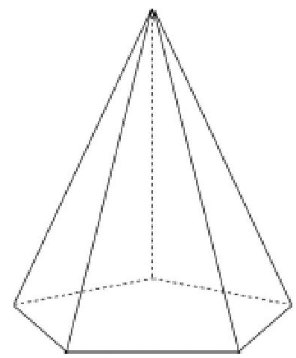
Edges ____



Lesson 7: General Pyramids and Cones and Their Cross-Sections

Opening Exercise

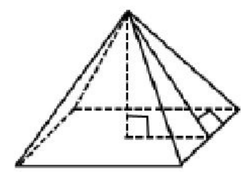




a. Draw the cross section for each figure shown, what similarities do they have?

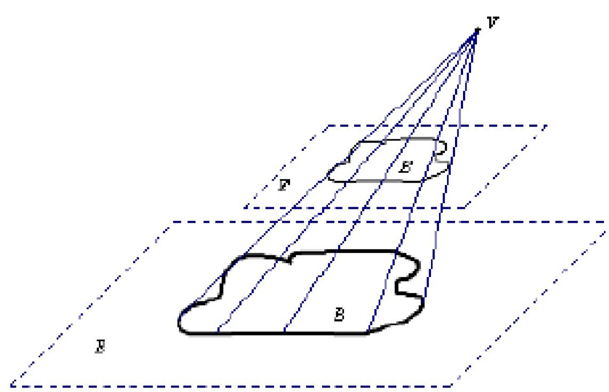
b. What differences do the cross sections have?

Vocabulary

Figure	Description	Diagram
General Cone	A solid figure with a curved or semi-curved base. A general cone has a vertex, just like a pyramid.	
Pyramid	A general cone with a polygonal base.	

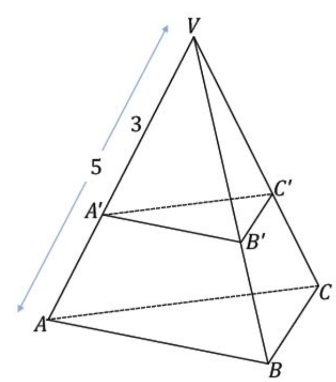
Cross sections of cones

The cross section of a figure is a slice that is taken ____________________ to the base of the solid. In a general cone, the cross section will always be a figure that is _____________________to the base of the cone. The vertex of the cone can then be thought of as the center of dilation.



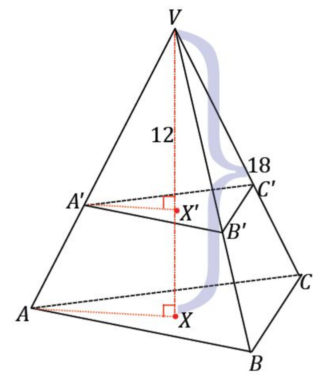
Example 1

In the following triangular pyramid, a plane passes through the pyramid so that it is parallel to the base and results in the cross-section $\triangle A'B'C'$. If the area of $\triangle ABC$ is $25\ mm^2$, what is the area of $\triangle A'B'C'$?



Example 2

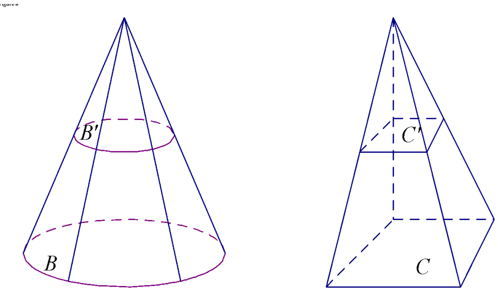
In the following triangular pyramid, a plane passes through the pyramid so that it is parallel to the base and results in the cross-section $\triangle A'B'C'$. The altitude from V is drawn; the intersection of the altitude with the base is X, and the intersection of the altitude with the cross-section is X'. If the distance from X to V is $18\ mm$, the distance from X' to V is $12\ mm$, and the area of $\triangle A'B'C'$ is $28\ mm^2$, what is the area of $\triangle ABC$?



Example 3

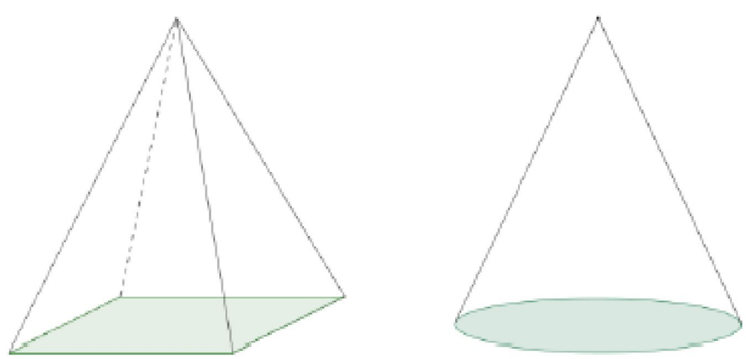
GENERAL CONE CROSS-SECTION THEOREM: If two general cones have the same base area and the same height, then cross-sections for the general cones the same distance from the vertex have the same area.

State the theorem in your own words:



Example 4

The diagram below shows a circular cone and a general pyramid. The bases of the cones are equal in area, and the solids have equal heights.



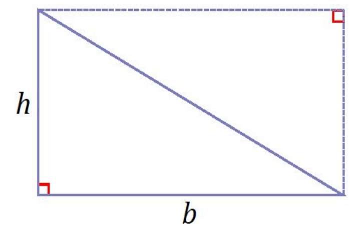
a. Sketch a slice in each cone that is parallel to the base and 2/3 closer to the vertex than the base of the cone.

b. If the area of the base of the circular cone is $616u^2$, find the exact area of the slice taken from the pyramid.

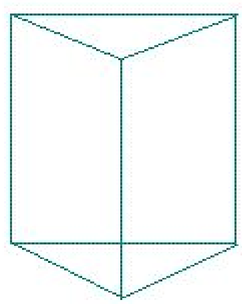
Lesson 8: Definition and Properties of Volume

Opening Exercise

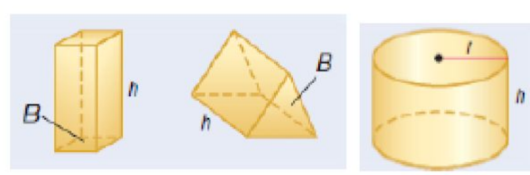
a. Use the following image to reason why the area of a right triangle is $\frac{1}{2}bh$ (Area Property 2).



b. Use the following image to reason why the Volume of a triangular prism is v=Bh where B is the area of the base and h is the height of the prism:



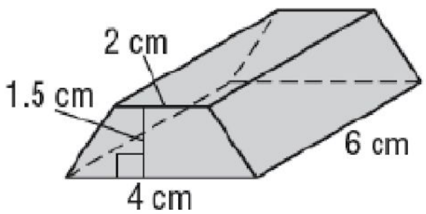
We can think of calculating volume by stacking the area of a shape until we get our desired height

Name	Formula	Diagram
Volume of a General Cylinder	$V = Bh$ where B is the area of the base and h is the height	

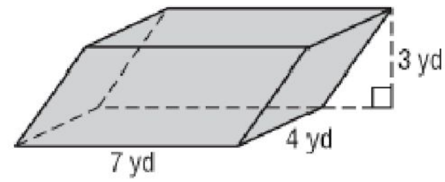
Exercises

Find the volume of the following prisms:

1.

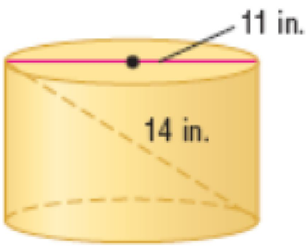


2.



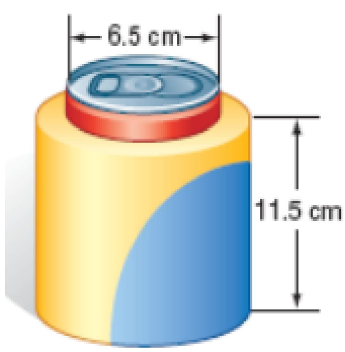
Example 1

Find the volume of the pictured circular cylinder to the nearest hundredth.



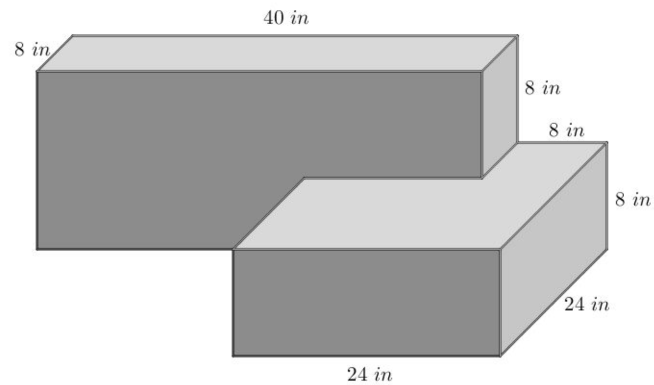
Example 2

A can 12 centimeters tall fits into a rubberized cylindrical holder that is 11.5 cm tall, including 1 cm for the thickness of the base of the holder. This thickness of the rim of the holder is 1 cm. What is the volume of the rubberized material that makes up the holder to the nearest hundredth?



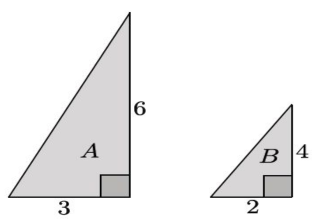
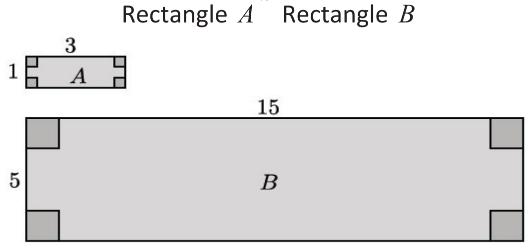
Example 3

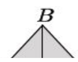
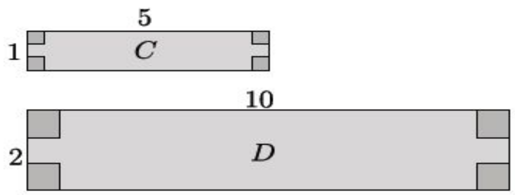
Paul is designing a mold for a concrete block to be used in a custom landscaping project. The block is shown in the diagram with its corresponding dimensions and consists of two intersecting rectangular prisms. Find the volume of mixed concrete, in cubic feet, needed to make Paul's custom block.



Lesson 9: Scaling Principle for Volumes

For each pair of similar figures, write the ratio of side lengths $a:b$ or $c:d$ that compares one pair of corresponding sides. Then, complete the third column by writing the ratio that compares the areas of the similar figures. Simplify ratios when possible.

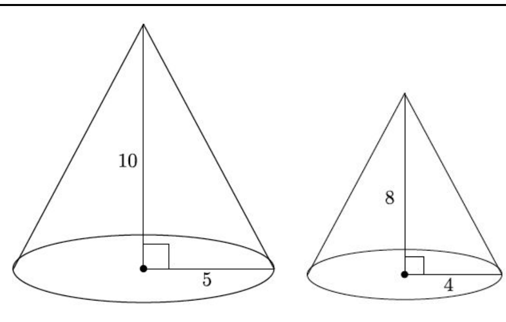
Similar Figures	**Ratio of Side Lengths** $a:b$ **or** $c:d$	**Ratio of Areas** $Area(A):Area(B)$ **or** $Area(C):Area(D)$
$\triangle A \sim \triangle B$ 	6 : 43 : 2	9 : 43^2 : 2^2
Rectangle $A \sim$ Rectangle B 		
$\triangle C \sim \triangle D$		

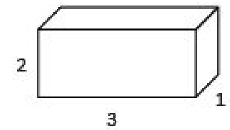
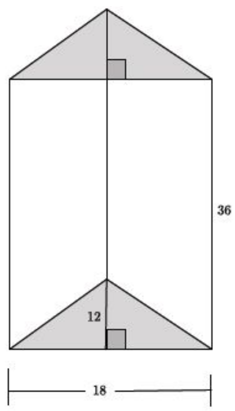
$\triangle A \sim \triangle B$ 		
Rectangle $A \sim$ Rectangle B 		

1.) State the relationship between the ratio of sides $a:b$ and the ratio of the areas $Area(A):Area(B)$.

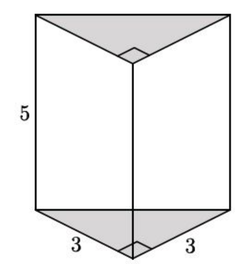
2.) Make a conjecture as to how the ratio of sides $a:b$ will be related to the ratio of volumes $Volume(S):Volume(T)$. Explain.

3.) Each pair of solids shown below is similar. Write the ratio of side lengths $a:b$ comparing one pair of corresponding sides. Then, complete the third column by writing the ratio that compares volumes of the similar figures. Simplify ratios when possible.

Similar Figures	**Ratio of Side Lengths** $a:b$	**Ratio of Volumes** $Volume(A):Volume(B)$
 **Figure A** **Figure B**		

 **Figure A**  **Figure B**		
 **Figure A** **Figure B**		

4.) Use the triangular prism shown to answer the questions that follow.



a. Calculate the volume of the triangular prism.

b. If one side of the triangular base is scaled by a factor of 2, the other side of the triangular base is scaled by a factor of 4, and the height of the prism is scaled by a factor of 3, what are the dimensions of the scaled triangular prism?

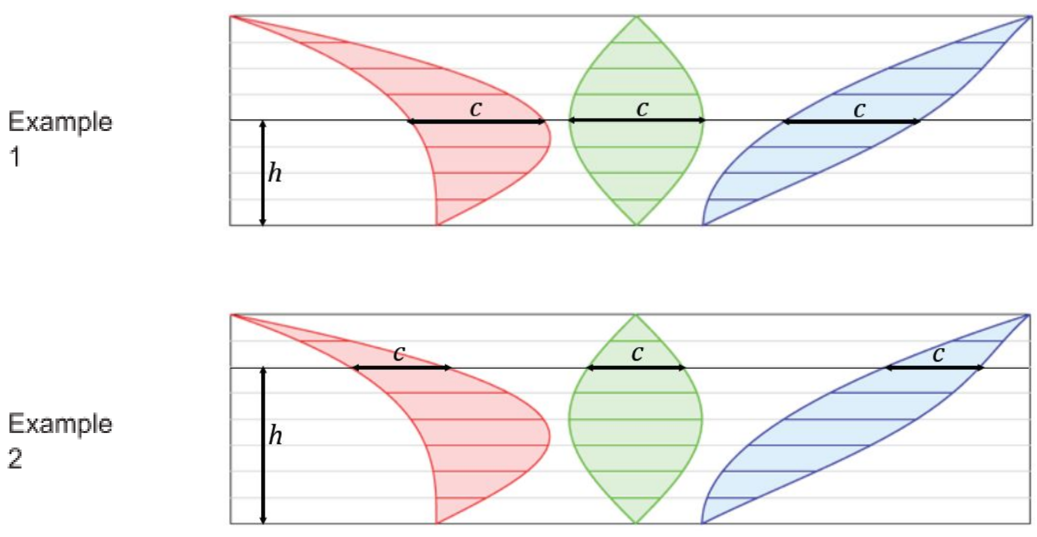
c. Calculate the volume of the scaled triangular prism.

d. Make a conjecture about the relationship between the volume of the original triangular prism and the scaled triangular prism.

e. Do the volumes of the figures have the same relationship as was shown in the figures in Exercise 1? Explain.

Lesson 10: The Volume of Prisms and Cylinders and Cavalieri's Principle

THEOREM: If two planar figures of equal altitude have identical cross-sectional lengths at each height, then the regions of the figures have the same area.



CAVALIERI'S PRINCIPLE: If two solids have the same height, every plane parallel to the base that intersects both solids in cross-sections of equal area, then the volumes of the two solids are equal.

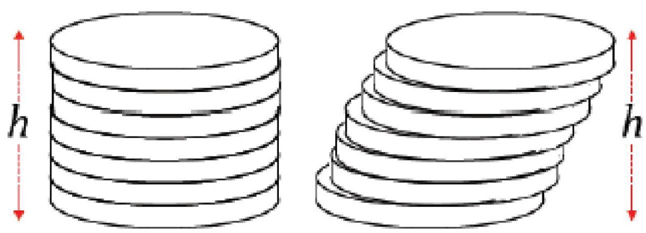
Figure 3

Figure 5

Check out the video tutorial! https://schoolyourself.org/learn/geometry/cavalieri-3d

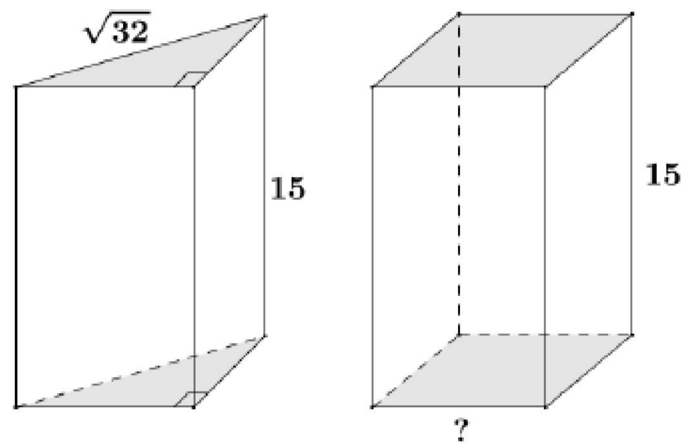
Example 1:

Looking at the stack of quarters below, what do we know about their volumes? Explain why.



Example 2

A triangular prism has an isosceles right triangular base with a hypotenuse of $\sqrt{32}$ and a prism height of 15. A square prism has a height of 15 and its volume is equal to that of the rectangular prism. What are the dimensions of the square base, in simplest radical form?

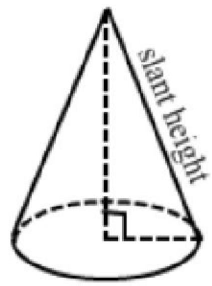
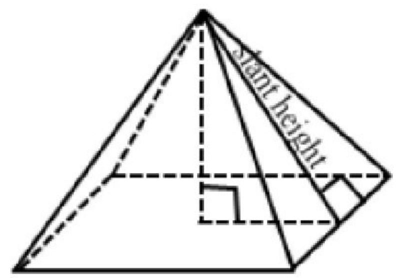


Example 3

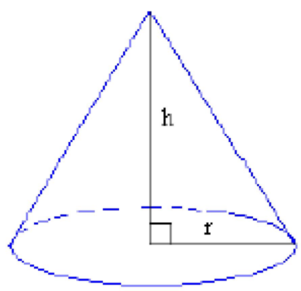
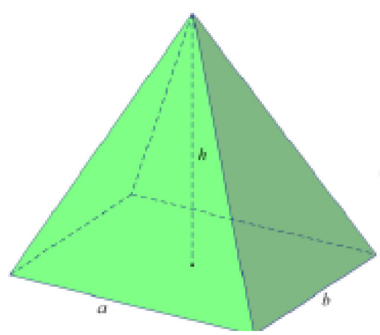
Find the volume of an oblique circular cylinder that has a radius of 5 feet and a height of 3 feet. Round to the nearest tenth.

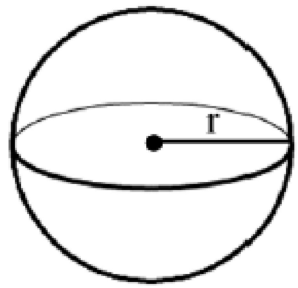
Lesson 11: The Volume Formula of a Pyramid and Cone

Vocabulary

Term	Definition	Diagram
Slant Height	The distance measured along a lateral face from the base to the vertex of a pyramid or cone. In the case of a pyramid, the slant height is the height of the triangular lateral face.	 

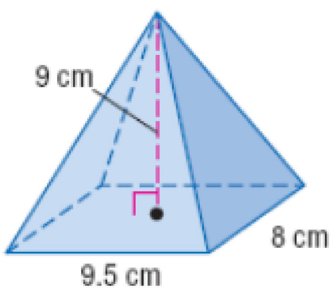
Volume Formulas:

Shape	Diagram	Formula
Cone		$V = \frac{1}{3}Bh$ *Where B is the area of the base*
Pyramid		$V = \frac{1}{3}Bh$ *Where B is the area of the base*

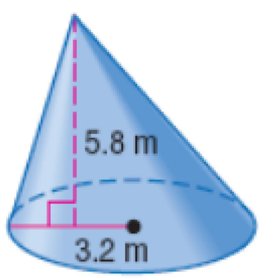
Sphere		$V = \frac{4}{3}\pi r^3$ *Where r is the radius*

Examples:

1.) Find the volume of the pyramid pictured below:



2.) Find the volume of the cone, leave your answer in terms of π.



3.) If the slant height of a cone is 26 and the radius of the base of the cone is 10, find the volume of the cone.

4.) An ice cream cone is 11cm deep and 5 cm across the opening of the cone. Two hemisphere-shaped scoops of ice cream, which also have a diameter of 5cm are placed on top of the cone. If the ice cream were te melt into the cone, will it overflow?