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7-3 Reteaching

Logarithmic Functions as Inverses

A logarithmic function is the inverse of an exponential function.

To evaluate logarithmic expressions, use the fact that $x = \log_b y$ is the same as $y = b^x$. Keep in mind that $x = \log y$ is another way of writing $x = \log_{10} y$.

Problem

What is the logarithmic form of $6^3 = 216$?

Step 1 Determine which equation to use.

The equation is in the form $b^x = y$.

Step 2 Find x, y, and b.

$b = 6$, $x = 3$, and $y = 216$

Step 3 Because $y = b^x$ is the same as $x = \log_b y$, rewrite the equation in logarithmic form by substituting for x, y, and b.

$3 = \log_6 216$

Exercises

Write each equation in logarithmic form.

1. $4^{-3} = \frac{1}{64}$
   $\log_4 \frac{1}{64} = -3$
2. $5^{-2} = \frac{1}{25}$
   $\log_5 \frac{1}{25} = -2$
3. $8^{-1} = \frac{1}{8}$
   $\log_8 \frac{1}{8} = -1$
4. $11^0 = 1$
   $\log_{11} 1 = 0$
5. $6^1 = 6$
   $\log_6 6 = 1$
6. $6^{-3} = \frac{1}{216}$
   $\log_6 \frac{1}{216} = -3$
7. $17^0 = 1$
   $\log_{17} 1 = 0$
8. $17^1 = 17$
   $\log_{17} 17 = 1$

Problem

What is the exponential form of $4 = \log_5 625$?

Step 1 Determine which equation to use.

The equation is in the form $x = \log_b y$.

Step 2 Find x, y, and b.

$x = 4$, $b = 5$, and $y = 625$

Step 3 Because $x = \log_b y$ is the same as $y = b^x$, rewrite the equation in exponential form by substituting for x, y, and b.

$625 = 5^4$

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7-3 Reteaching (continued)

Logarithmic Functions as Inverses

Exercises

Write each equation in exponential form.

9. $3 = \log_2 8$ $2^3 = 8$

10. $2 = \log_5 25$ $5^2 = 25$

11. $\log 0.1 = -1$ $10^{-1} = 0.1$

12. $\log 7 \approx 0.845$ $10^{0.845} \approx 7$

13. $\log 1000 = 3$ $10^3 = 1000$

14. $-2 = \log 0.01$ $10^{-2} = 0.01$

15. $\log_3 81 = 4$ $3^4 = 81$

16. $\log_{49} 7 = \frac{1}{2}$ $49^{\frac{1}{2}} = 7$

17. $\log_8 \frac{1}{4} = -\frac{2}{3}$ $8^{-\frac{2}{3}} = \frac{1}{4}$

18. $\log_2 128 = 7$ $2^7 = 128$

19. $\log_5 \frac{1}{625} = -4$ $5^{-4} = \frac{1}{625}$

20. $\log_6 36 = 2$ $6^2 = 36$

Problem

What is the value of $\log_4 32$?

$x = \log_4 32$	Write the equation in logarithmic form $x = \log_b y$.
$32 = 4^x$	Rewrite in exponential form $y = b^x$.
$2^5 = (2^2)^x$	Rewrite each side of the equation with like bases in order to solve the equation.
$2^5 = 2^{2x}$	Simplify.
$5 = 2x$	Set the exponents equal to each other.
$x = \frac{5}{2}$	Solve for x.

$\log_4 32 = \frac{5}{2}$

Exercises

Evaluate the logarithm.

21. $\log_2 64$ 6

22. $\log_4 64$ 3

23. $\log_3 3^4$ 4

24. $\log 10$ 1

25. $\log 0.1$ −1

26. $\log 1$ 0

27. $\log_8 2$ $\frac{1}{3}$

28. $\log_{32} 2$ $\frac{1}{5}$

29. $\log_9 3$ $\frac{1}{2}$

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7-4 Reteaching

Properties of Logarithms

You can write a logarithmic expression containing more than one logarithm as a single logarithm as long as the bases are equal. You can write a logarithm that contains a number raised to a power as a logarithm with the power as a coefficient. To understand the following properties, remember that logarithms are powers.

Name	Formula	Why?
Product Property	$\log_b mn = \log_b m + \log_b n$	When you multiply two powers, you add the exponents. Example: $2^6 \cdot 2^2 = 2^{(6+2)} = 2^8$
Quotient Property	$\log_b \frac{m}{n} = \log_b m - \log_b n$	When you divide two powers, you subtract the exponents. Example: $\frac{2^6}{2^2} = 2^{(6-2)} = 2^4$
Power Property	$\log_b m^n = n\log_b m$	When you raise a power to a power, you multiply the exponents. Example: $(2^6)^2 = 2^{(6 \cdot 2)} = 2^{12}$

Problem

What is $2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27$ written as a single logarithm?

$2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27 = \log_2 6^2 - \log_2 9 + \log_2 27^{\frac{1}{3}}$ Use the Power Property twice.

$= \log_2 36 - \log_2 9 + \log_2 3$ $6^2 = 36$, $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

$= (\log_2 36 - \log_2 9) + \log_2 3$ Group two of the logarithms. Use order of operations.

$= \log_2 \frac{36}{9} + \log_2 3$ Quotient Property

$= \log_2 \left(\frac{36}{9} \cdot 3\right)$ Product Property

$= \log_2 12$ Simplify.

As a single logarithm, $2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27 = \log_2 12$.

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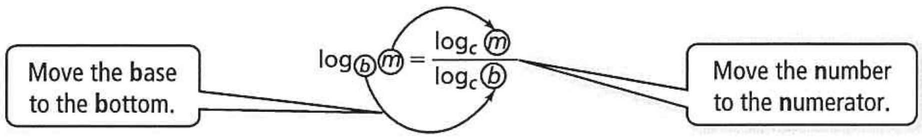
7-4 Reteaching (continued)

Properties of Logarithms

To evaluate logarithms with any base, you can rewrite the logarithm as a quotient of two logarithms with the same base.



Problem

What is $\log_4 8$ written as a quotient of two logarithms with base 2? Simplify your answer, if possible.

$\log_4 8 = \frac{\log_2 8}{\log_2 4}$ The base is 4 and the number is 8. Move the base to the bottom and the number to the numerator.

$= \frac{3}{2}$ Evaluate the logarithms in the numerator and the denominator.

Exercises

Write each logarithmic expression as a single logarithm.

1. $\log_3 13 + \log_3 3$ **$\log_3 39$**
2. $2\log x + \log 5$ **$\log 5x^2$**
3. $\log_4 2 - \log_4 6$ **$\log_4 \frac{1}{3}$**
4. $3\log_3 3 - \log_3 3$ **$\log_3 9$, or 2**
5. $\log_5 8 + \log_5 x$ **$\log_5 8x$**
6. $\log 2 - 2\log x$ **$\log \frac{2}{x^2}$**
7. $\log_2 x + \log_2 y$ **$\log_2 xy$**
8. $3\log_7 x - 5\log_7 y$ **$\log_7 \frac{x^3}{y^5}$**
9. $4\log x + 3\log x$ **$\log x^7$**
10. $\log_5 x + 3\log_5 y$ **$\log_5 xy^3$**
11. $3\log_2 x - \log_2 y$ **$\log_2 \frac{x^3}{y}$**
12. $\log_2 16 - \log_2 8$ **$\log_2 2$, or 1**

Write each logarithm as a quotient of two common logarithms. Simplify your answer, if possible. (*Hint:* Common logarithms are logarithms with base 10.)

13. $\log_4 12$ **$\frac{\log 12}{\log 4}$**
14. $\log_2 1000$ **$\frac{3}{\log 2}$**
15. $\log_5 16$ **$\frac{\log 16}{\log 5}$**
16. $\log_{11} 205$ **$\frac{\log 205}{\log 11}$**
17. $\log_9 32$ **$\frac{\log 32}{\log 9}$**
18. $\log_{100} 51$ **$\frac{\log 51}{2}$**

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7-4 Practice

Form K

Properties of Logarithms

Properties of Logarithms		
Product Property	**Quotient Property**	**Power Property**
$\log_b mn = \log_b m + \log_b n$	$\log_b \frac{m}{n} = \log_b m - \log_b n$	$\log_b m^n = n \log_b m$

Write each expression as a single logarithm.

1. $\log_3 9 + \log_3 24$ **$\log_3 216$**
2. $\log_4 16^3$ **$3 \log_4 16$**
3. $\log_2 7 - \log_2 9$ **$\log_2 \frac{7}{9}$**
4. $\log_3 8^5$ **$5 \log_3 8$**
5. $\log_4 x - \log_4 y$ **$\log_4 \frac{x}{y}$**
6. $\log 5 + \log 7$ **$\log 35$**

Expand each logarithm. Simplify if possible.

7. $\log_3 27x$ **$\log_3 27 + \log_3 x$**
8. $\log \frac{3}{7}$ **$\log 3 - \log 7$**
9. $\log_4 y^2z^3$ **$2 \log_4 y + 3 \log_4 z$**
10. $\log_5 \frac{3^2}{x}$ **$2 \log_5 3 - \log_5 x$**
11. $\log_3 15xy$ **$1 + \log_3 5 + \log_3 x + \log_3 y$**
12. $\log 8xz^4$ **$3 \log 2 + \log x + 4 \log z$**

13. **Open-Ended** Write three different logarithms. You should be able to expand each logarithm by one of the properties of logarithms. **Answers may vary. Sample: $\log_5 6x$, $\log \frac{3}{4}$, $\log_3 7^2$**

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7-4 Practice (continued)

Form K

Properties of Logarithms

Change of Base Formula
For any positive numbers m, b, and c, with $b \neq 1$ and $c \neq 1$, $$\log_b m = \frac{\log_c m}{\log_c b}$$

Use the Change of Base Formula to evaluate each expression.

14. $\log_{32} 4$

$\frac{\log_2 4}{\log_2 32} = \frac{2}{5} = 0.4$

15. $\log_9 27$ **1.5**

16. $\log_4 12$ **about 1.792**

17. Error Analysis Your friend used the Change of Base Formula to evaluate the expression $\log_4 8$. Her answer was $\frac{2}{3}$. What error did your friend make? What is the correct answer?
Sample answer: Your friend confused the numerator and the denominator in the formula. The correct answer is $\frac{3}{2}$.

Use the following formula to solve Exercise 18.

Formula for Loudness of a Sound (decibels)
$$L = 10 \log \frac{I}{I_0}$$ • I is the intensity of a sound in watts per square meter (W/m^2). • I_0 is the intensity of a sound that can barely be heard. • $I_0 = 10^{-12}\ W/m^2$

18. Your classmate went to a rock concert. At the loudest point during the concert, the sound had an intensity of $2.35 \times 10^{-3}\ W/m^2$. What was the loudness of this sound in decibels?
about 93.71 decibels

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7-5 Reteaching

Exponential and Logarithmic Equations

Use logarithms to solve exponential equations.

Problem

What is the solution of $7 - 5^{2x-1} = 4$?

$7 - 5^{2x-1} = 4$	
$-5^{2x-1} = -3$	First isolate the term that has the variable in the exponent. Begin by subtracting 7 from each side.
$5^{2x-1} = 3$	Multiply each side by -1.
$\log_5 5^{2x-1} = \log_5 3$	Because the variable is in the exponent, use logarithms. Take $\log_5$ of each side because 5 is the base of the exponent.
$(2x - 1)\log_5 5 = \log_5 3$	Use the Power Property of Logarithms.
$2x - 1 = \log_5 3$	Simplify. (Recall that $\log_b b = 1$.)
$2x - 1 = \frac{\log 3}{\log 5}$	Apply the Change of Base Formula.
$2x = \frac{\log 3}{\log 5} + 1$	Add 1 to each side.
$x = \frac{1}{2}\left(\frac{\log 3}{\log 5} + 1\right)$	Divide each side by 2.
$x \approx 0.84$	Use a calculator to find a decimal approximation.

Exercises

Solve each equation. Round the answer to the nearest hundredth.

1. $2^x = 5$ **2.32**

2. $10^{2x} = 8$ **0.45**

3. $5^{x+1} = 25$ **1**

4. $2^{x+3} = 9$ **0.17**

5. $3^{2x-3} = 7$ **2.39**

6. $4^x - 5 = 3$ **1.50**

7. $5 + 2^{x+6} = 9$ **−4**

8. $4^{3x} + 2 = 3$ **0**

9. $1 - 3^{2x} = -5$ **0.82**

10. $2^{3x} - 2 = 13$ **1.30**

11. $5^{2x+7} - 1 = 8$ **−2.82**

12. $7 - 2^{x+7} = 5$ **−6**

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7-5 Reteaching (continued)

Exponential and Logarithmic Equations

Use exponents to solve logarithmic equations.

Problem

What is the solution of $8 - \log(4x - 3) = 4$?

$8 - \log(4x - 3) = 4$	
$-\log(4x - 3) = -4$	First isolate the term that has the variable in the logarithm. Begin by subtracting 8 from each side.
$\log(4x - 3) = 4$	Multiply each side by –1.
$4x - 3 = 10^4$	Write in exponential form.
$4x - 3 = 10{,}000$	Simplify.
$4x = 10{,}003$	Add 3 to each side.
$x = \frac{10{,}003}{4}$	Solve for x.
$x = 2500.75$	Divide.

Exercises

Solve each equation. Round the answer to the nearest thousandth.

13. $\log x = 2$ **100**

14. $\log 3x = 3$ **333.333**

15. $\log 2x + 2 = 6$ **5000**

16. $5 + \log(2x + 1) = 6$ **4.5**

17. $\log 5x + 62 = 62$ **0.2**

18. $6 - \log\frac{1}{2}x = 3$ **2000**

19. $\log(4x - 3) + 6 = 4$ **0.753**

20. $\frac{2}{3}\log 5x = 2$ **200**

21. $2\log 250x - 6 = 4$ **400**

22. $5 - 2\log x = \frac{1}{2}$ **177.828**

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7-5 Practice

Form K

Exponential and Logarithmic Equations

Solve each equation. To start, rewrite each side with a common base.

1. $125^{2x} = 25$

$(5^3)^{2x} = 5^2$

$5^{6x} = 5^2$

$6x = 2$

$x = \frac{1}{3}$

2. $2^{3x-3} = 64$

$2^{3x-3} = 2^6$

$x = 3$

3. $81^{3x} = 27$

$x = \frac{1}{4}$

Solve each equation. Round to the nearest ten-thousandth. Check your answers. To start, take the logarithm of each side.

4. $6^{4x} = 234$

$\log 6^{4x} = \log 234$

$4x \log 6 = \log 234$

$x = \frac{\log 234}{4 \log 6}$

$x \approx 0.7612$

5. $3^{5x} = 375$

$\log 3^{5x} = \log 375$

$x \approx 1.0790$

6. $7^{3x} - 24 = 184$

$x \approx 0.9143$

Graphing Calculator **Solve by graphing. Round to the nearest ten-thousandth.**

7. $3^{6x} = 2000$

Let $Y_1 = 3^{6x}$ and $Y_2 = 2000$.

$x \approx 1.1531$

8. $8^{3x} = 154$

$x \approx 0.8074$

9. $12^{4x} = 4600$

$x \approx 0.8485$

Use the following formula for Exercise 10.

$$T(m) = a(1 + r)^m$$

- m = the number of minutes it takes for $\frac{3}{4}$ of the crowd to leave the stadium
- $T(m)$ = the number of people in the stadium after m minutes
- a = the number of people currently in the stadium
- r = the percent change in the number of people in the stadium

10. There are currently 100,000 people in a stadium watching a soccer game. When the game ends, about 3% of the crowd will leave the stadium each minute. At this rate, how many minutes will it take for $\frac{3}{4}$ of the crowd to leave the stadium? **about 46 minutes**

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7-5 Practice (continued) *Form K*

Exponential and Logarithmic Equations

Convert from Logarithmic Form to Exponential Form to solve each equation.

Exponential and Logarithmic Form	
Logarithmic Form $\log_b x = y$	Exponential Form $b^y = x$

11. $\log(2x + 4) = 3$

$2x + 4 = 10^3$
$2x = 996$
$x = 498$

12. $\log 4z - 3 = 2$

$\log 4z = 5$
$z = 25{,}000$

13. $\log(2x - 8) = 2$

$x = 54$

Use the properties of logarithms to solve each equation.

Product Property	Quotient Property	Power Property
$\log_b mn = \log_b m + \log_b n$	$\log_b \frac{m}{n} = \log_b m - \log_b n$	$\log_b m^n = n \log_b m$

14. $2\log x + \log 4 = 3$

$\log x^2 + \log 4 = 3$
$\log 4x^2 = 3$
$4x^2 = 10^3$
$x^2 = 250$
$x \approx 15.81$

15. $\log y - \log 4 = 2$

$\log \frac{y}{4} = 2$
$y = 400$

16. $\log 10 + \log 2x = 3$

$x = 50$

17. Error Analysis Your friend used the following steps to solve the equation $\log x + \log 6 = 4$. What error did he make? What is the correct answer?

$\log x + \log 6 = 4$
$\log \frac{x}{6} = 4$
$\frac{x}{6} = 10^4$
$x = 6000$

He applied the Quotient Property rather than the Product Property; $x = \frac{5000}{3}$.

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Chapter 7 Find the Errors!

For use with Lessons 7-3 through 7-4

1. The base of the log is 6.

$$\begin{aligned} 6^5 &= 7776 \\ \log_6 6^5 &= \log_6 7776 \\ 5 &= \log_6 7776 \end{aligned}$$

2. The base of the log is 8.

$$\begin{aligned} \log_8 4 &= x \\ 8^{(\log_8 4)} &= 8^x \\ 4 &= 8^x \\ 2^2 &= 2^{3x} \\ 2 &= 3x \\ \frac{2}{3} &= x \end{aligned}$$

3. The exponent goes with the entire expression, not just the numerator.

$$\begin{aligned} 3\log x - 3\log 5 &= \log x^3 - \log 5^3 \\ &= \log \frac{x^3}{5^3} \\ &= \log \frac{x^3}{125} \end{aligned}$$

4. $\log(x+1)$ is not equal to $\log x + \log 1$.

$$\log \frac{(x+1)}{x} = \log(x+1) - \log x$$

5. The formula $\log \frac{I_1}{I_2} = M_1 - M_2$ compares the intensity levels of earthquakes.

Know
The magnitudes of the earthquakes.

Need
How many times more intense the first earthquake was.

Plan
Use the equation $\log \frac{I_1}{I_2} = M_1 - M_2$.

$$\begin{aligned} \log \frac{I_1}{I_2} &= M_1 - M_2 \\ \log \frac{I_1}{I_2} &= 6.9 - 5.4 \\ \log \frac{I_1}{I_2} &= 1.5 \\ \frac{I_1}{I_2} &= 10^{1.5} \\ &\approx 31.62 \end{aligned}$$

The 1989 earthquake was about 31.6 times more intense than the one in 2008.

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Chapter 7 Find the Errors!

For use with Lessons 7-5 through 7-6

1. 4^{x+3} should be rewritten as $2^{2(x+3)}$.

$$\begin{aligned} 4^{x+3} &= 16 \\ 2^{2(x+3)} &= 2^4 \\ 2(x+3) &= 4 \\ 2x+6 &= 4 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

2. The expression $\log 5x - \log 5$ was not simplified correctly.

$$\begin{aligned} \log 5x - \log 5 &= 2 \\ \log \frac{5x}{5} &= 2 \\ x &= 10^2 \\ x &= 100 \end{aligned}$$

3. The 3 cannot be divided out from within the natural logarithm.

$$\begin{aligned} 5 + \ln 3x &= 8 \\ \ln 3x &= 3 \\ 3x &= e^3 \\ x &= \frac{e^3}{3} \\ x &\approx 6.7 \end{aligned}$$

4. The natural logarithm should have been used.

$$\begin{aligned} 5e^x - 3 &= 7 \\ 5e^x &= 10 \\ e^x &= 2 \\ x &= \ln 2 \\ x &\approx 0.69 \end{aligned}$$

5. In simplifying $\frac{\log 0.3}{\log 0.93}$, the quotient of the logs should be found, not the log of the quotient.

If she starts with 100% of the land and sells 7%, she will have 93% of the land left. After n years, she will have $2000(0.93)^n$ of the land left.

$$\begin{aligned} 600 &= 2000(0.93)^n \\ 0.3 &= (0.93)^n \\ \log 0.3 &= \log 0.93^n \\ \log 0.3 &= n \log 0.93 \\ \frac{\log 0.3}{\log 0.93} &= n \\ 16.59 &\approx n \end{aligned}$$

In about 16.59 years, she will have 600 square miles of land left.

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